Towards Numerical Assistants

Trust, Measurement, Community, and Generality for the Numerical Workbench

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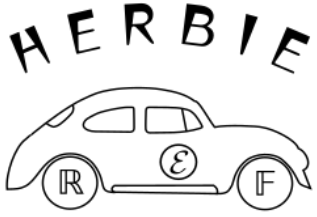


Writing Numerical Code

Diabolical errors

- Silent, non-compositional, non-local

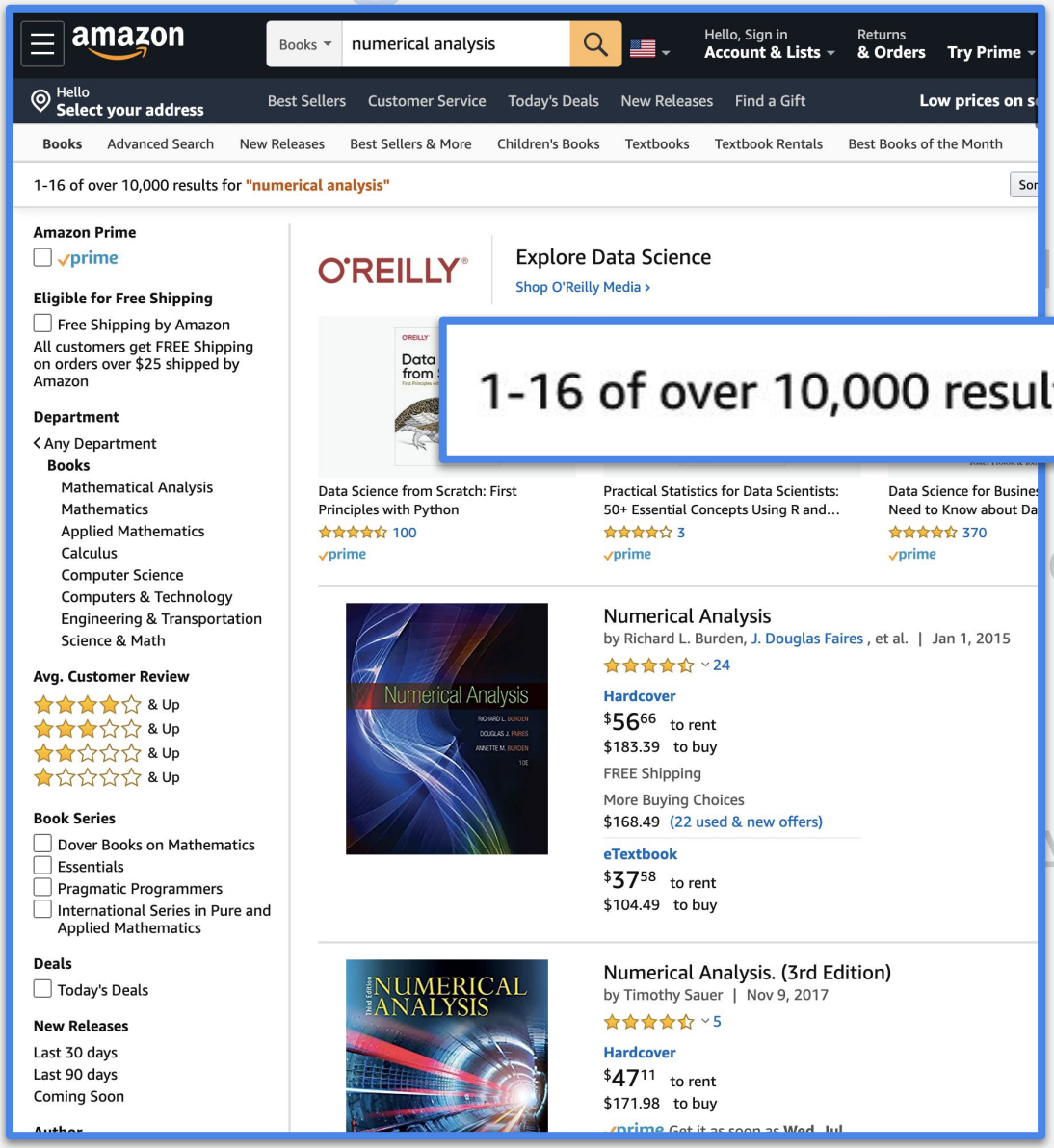
Subtle tradeoffs

- Performance, accuracy, determinism

Evolving landscape

- bfloat, posits, Flexpoint, AdaptivFloat, ...

Writing Numerical Code



l, non-local

1-16 of over 10,000 results for "numerical analysis"

determinism

AdaptivFloat, ...

Ideal Numerical Coding

1. Look up the formula you need from Wikipedia.
2. Type formula into your program.
3. Get accurate results.

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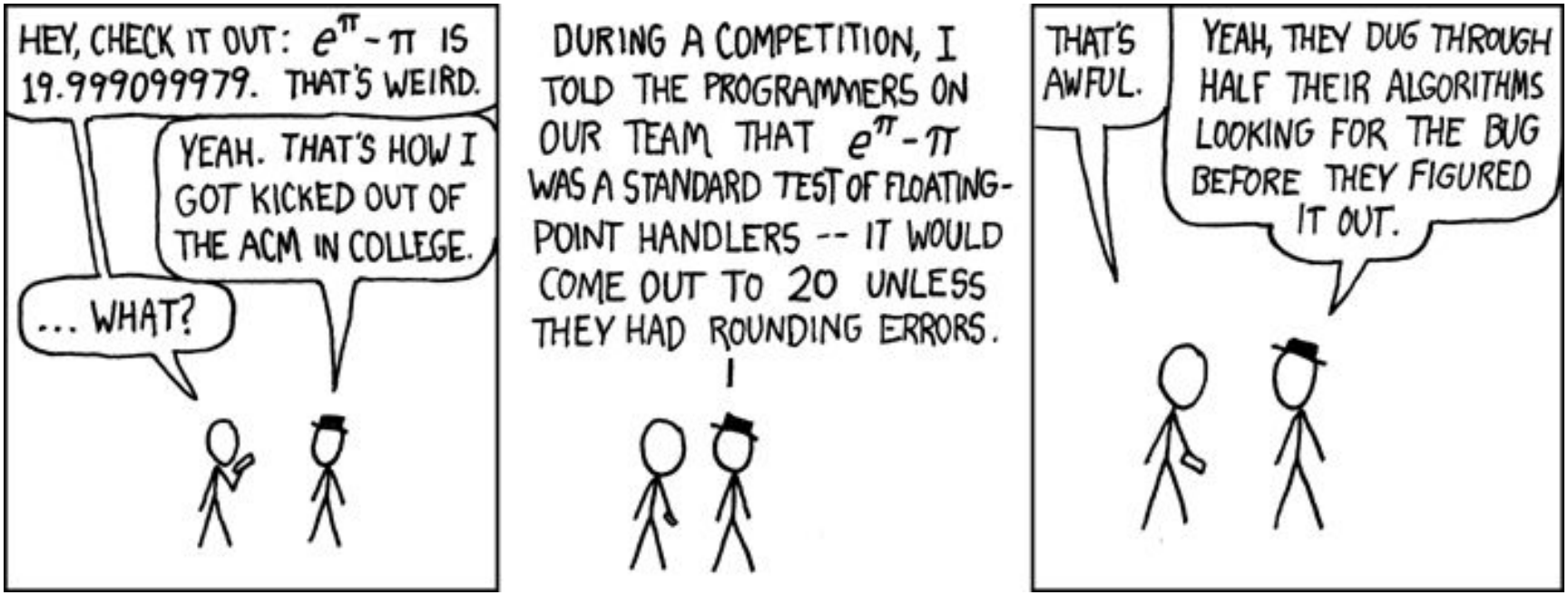

https://xkcd.com/217/

Ideal Numerical Coding

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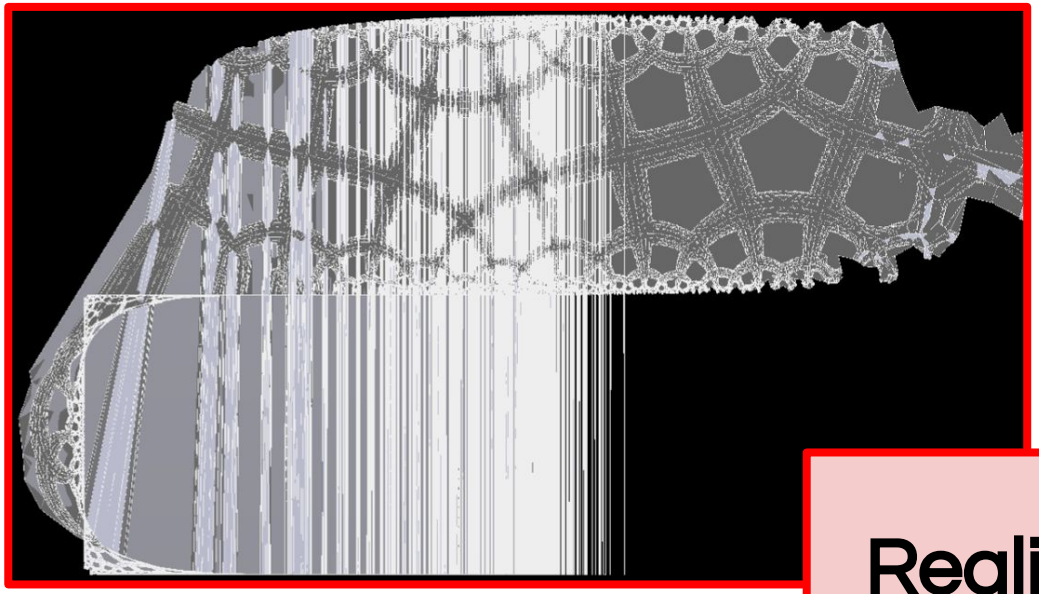
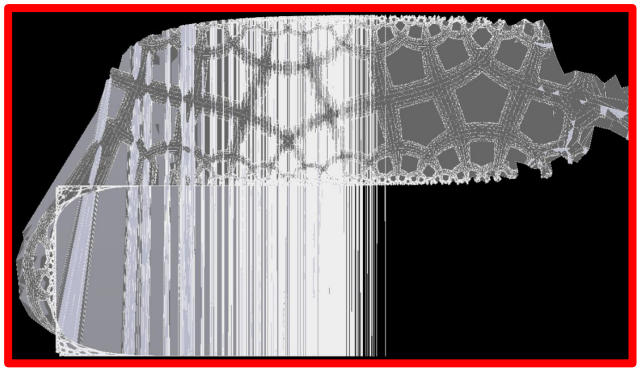


Reality.

Numerical Code in Practice

Blake Courter
CAD Researcher



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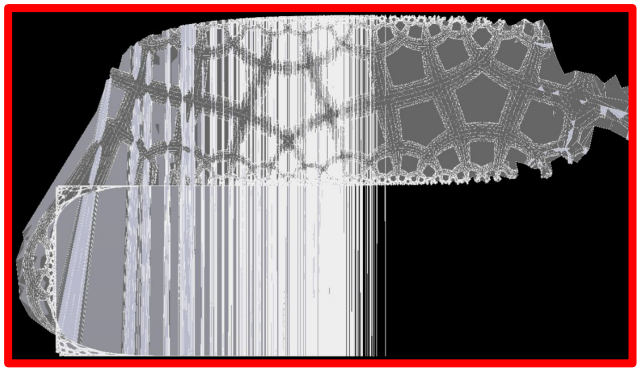

Vision: Numerical Assistants

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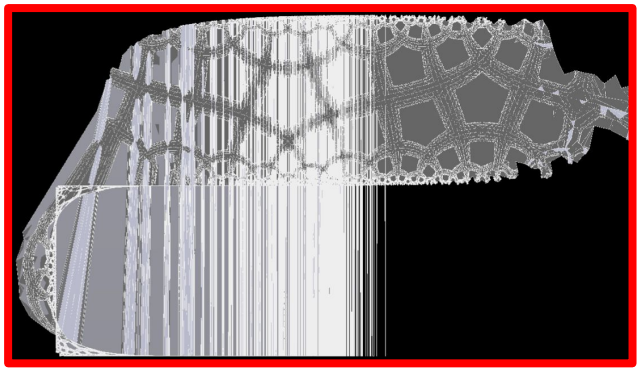

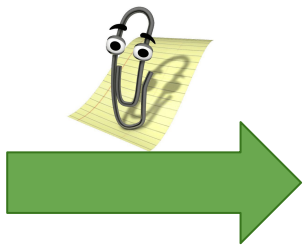

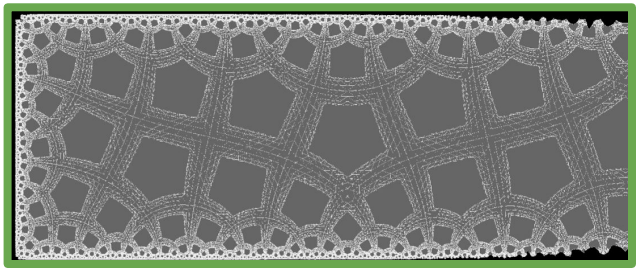

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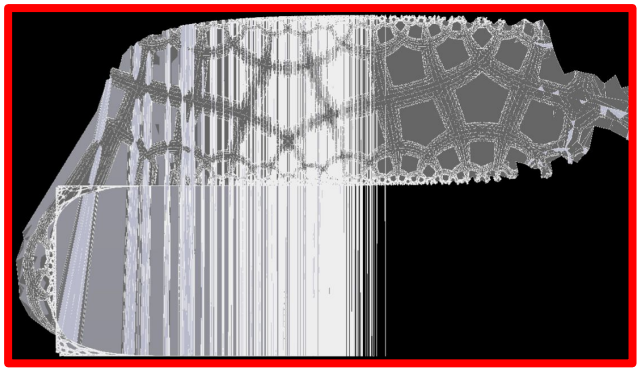

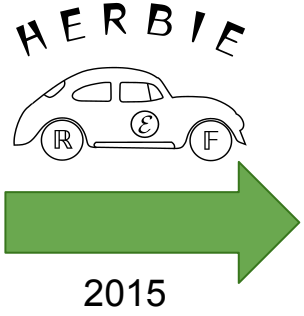

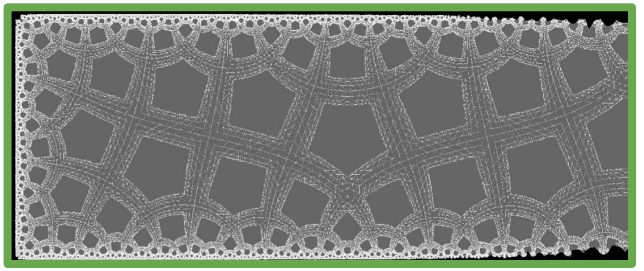

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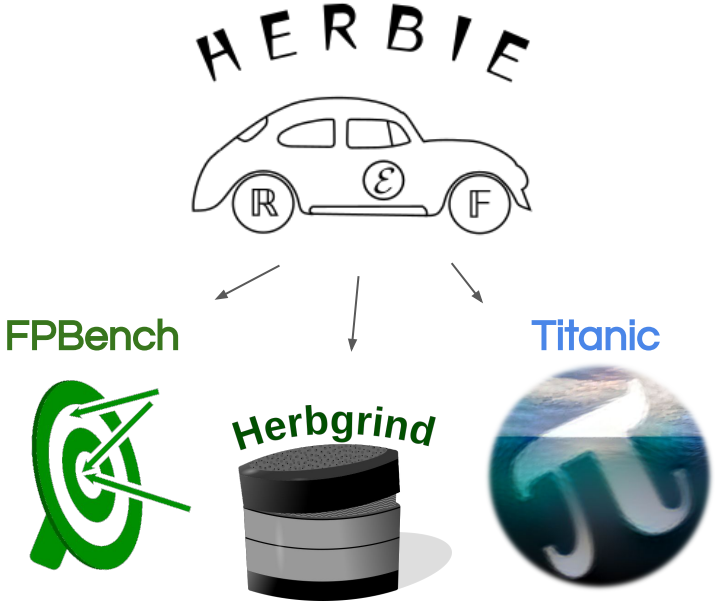

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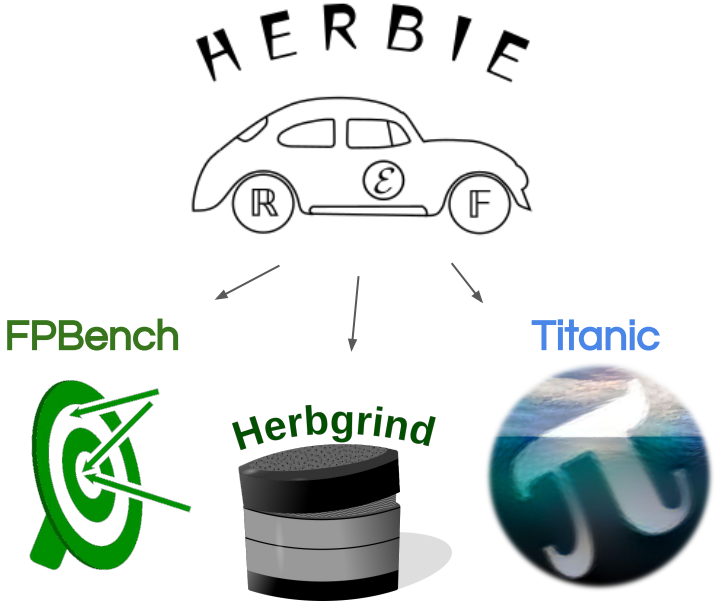
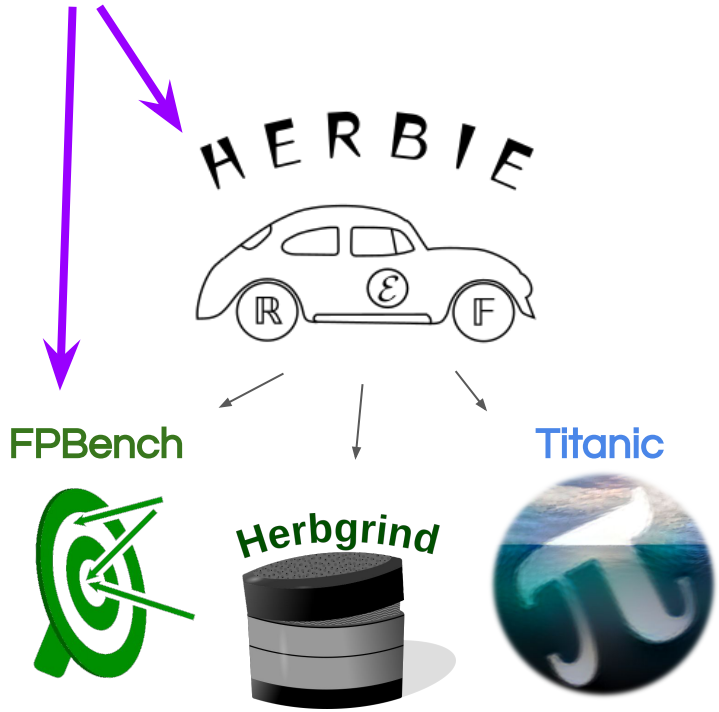


Developing since 2015:

1. Trust
2. Measurement
3. Community
4. Generality

Vision: Numerical Assistants

First, quick background



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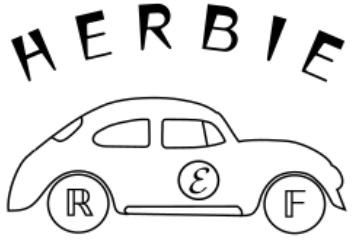
Herbie

Math

$$\frac{x - \sin x}{x - \tan x}$$



Herbie

Given input expression over the reals

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$$\frac{x - \sin x}{x - \tan x}$$

Herbie

Given input expression over the reals

Synthesize a floating point implementation

Math

$$\frac{x - \sin x}{x - \tan x}$$

$$\downarrow$$

$$\textbf{if } x \leq -0.03256340944027832 \vee \neg (x \leq 0.027721191804569126) :$$
$$\quad \frac{x}{x - \tan x} - \frac{\sin x}{x - \tan x}$$
$$\textbf{else} :$$
$$\quad x \cdot (x \cdot 0.225) + \left(-0.5 + x^4 \cdot -0.009642857142857142\right)$$

Herbie

Given input expression over the reals

Synthesize a floating point implementation

Less error than naive translation

Math

$$\frac{x - \sin x}{x - \tan x}$$

$$\downarrow$$

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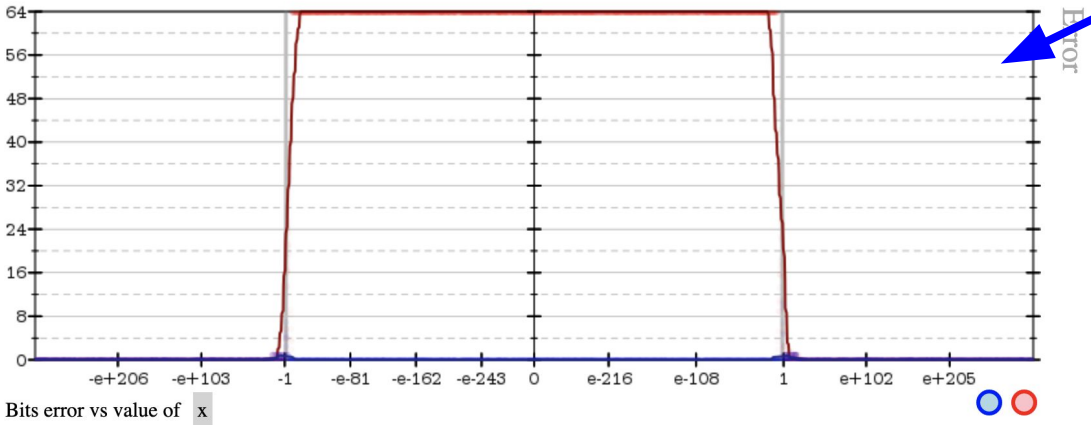
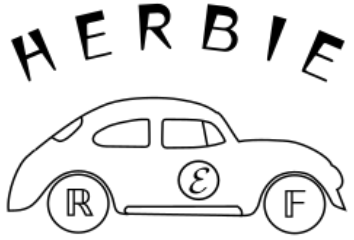


Bits error vs value of x



Herbie

“The drunk numerical analyst”

edwardkmett 35 points · 4 years ago

Having applied a few dozen of these changes now I can see how `herbie` will start to fit into my workflow.

If `hlint` is like having a drunken undergraduate do a code review, `herbie` is like having a drunken numerical analyst pore over every line of code you've written offering up suggestions for how to improve stability without much considering the surrounding context. While you probably have access to the former, the latter is a much rarer talent pool.

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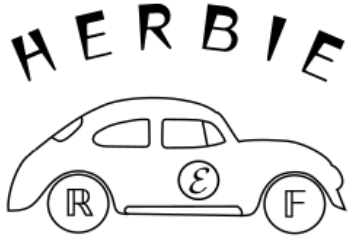
Live demo: **herbie.uwplse.org**



FPBench

Benchmarks

- Mine papers, enable apples-to-apples

Compilers

- Generate C, JavaScript, Scala, TeX, ...

Standards

- Support tool interchange, composition

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Live tour: **fpbench.org**

1. Trust

Why should users trust Herbie?

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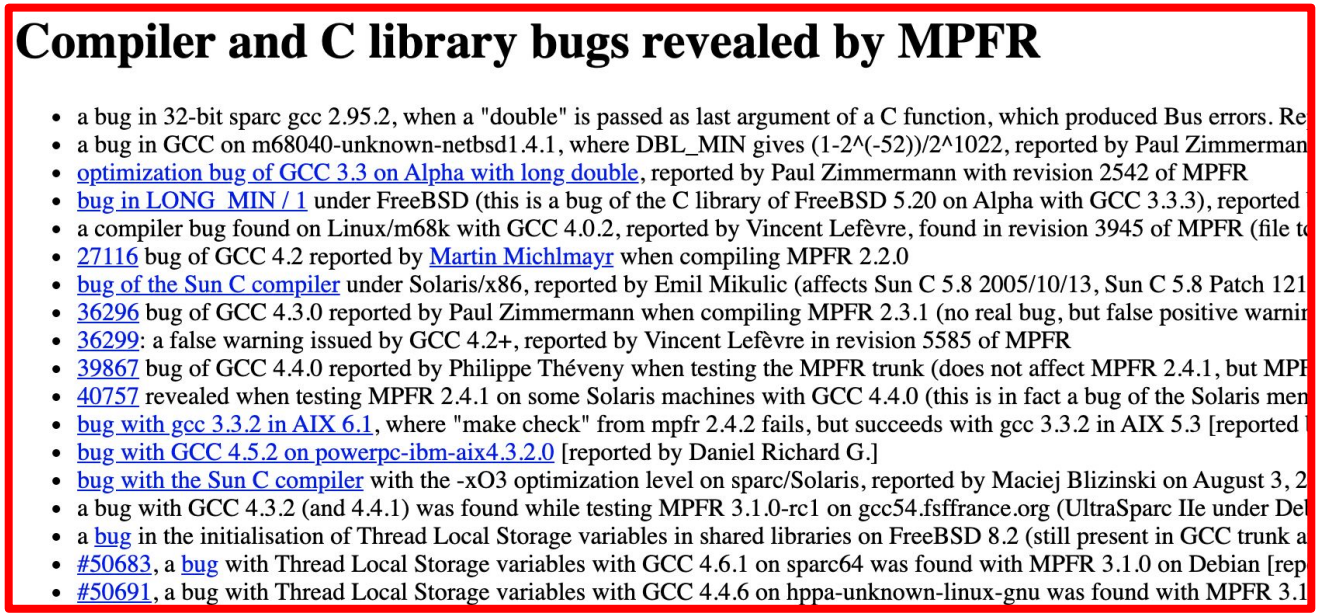
Compilers forbidden from touching floating point at all … and they even get that wrong!



Compiler and C library bugs revealed by MPFR

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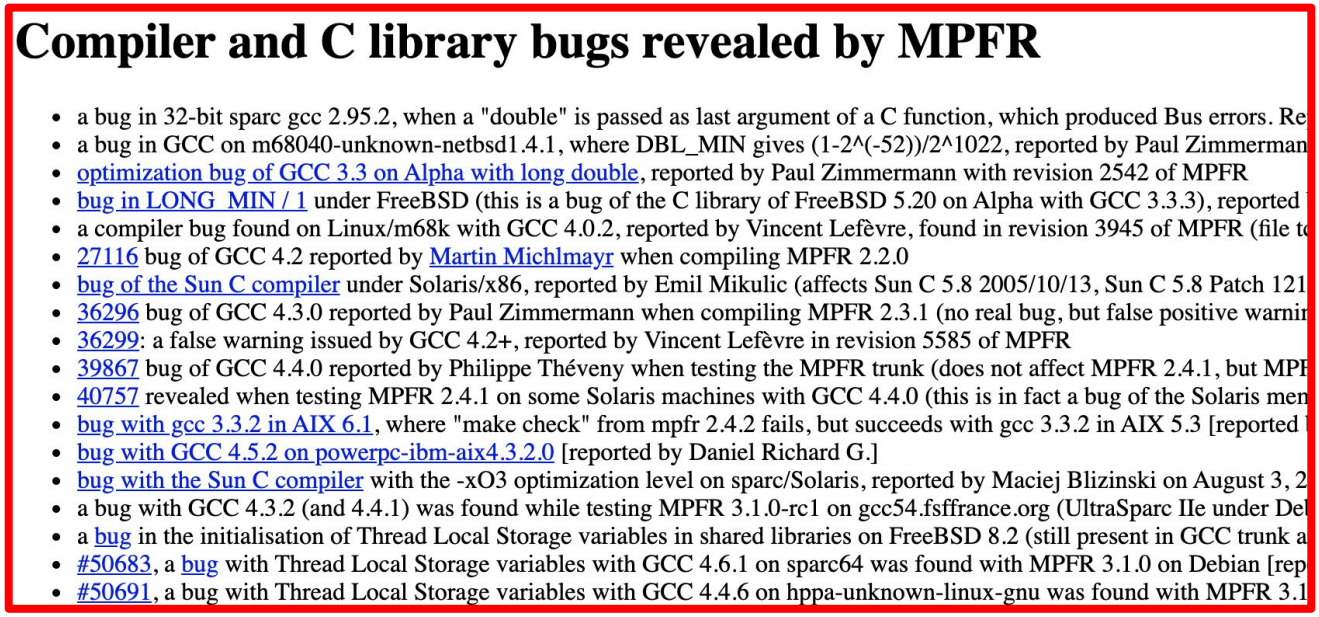

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No really, why should users trust Herbie?

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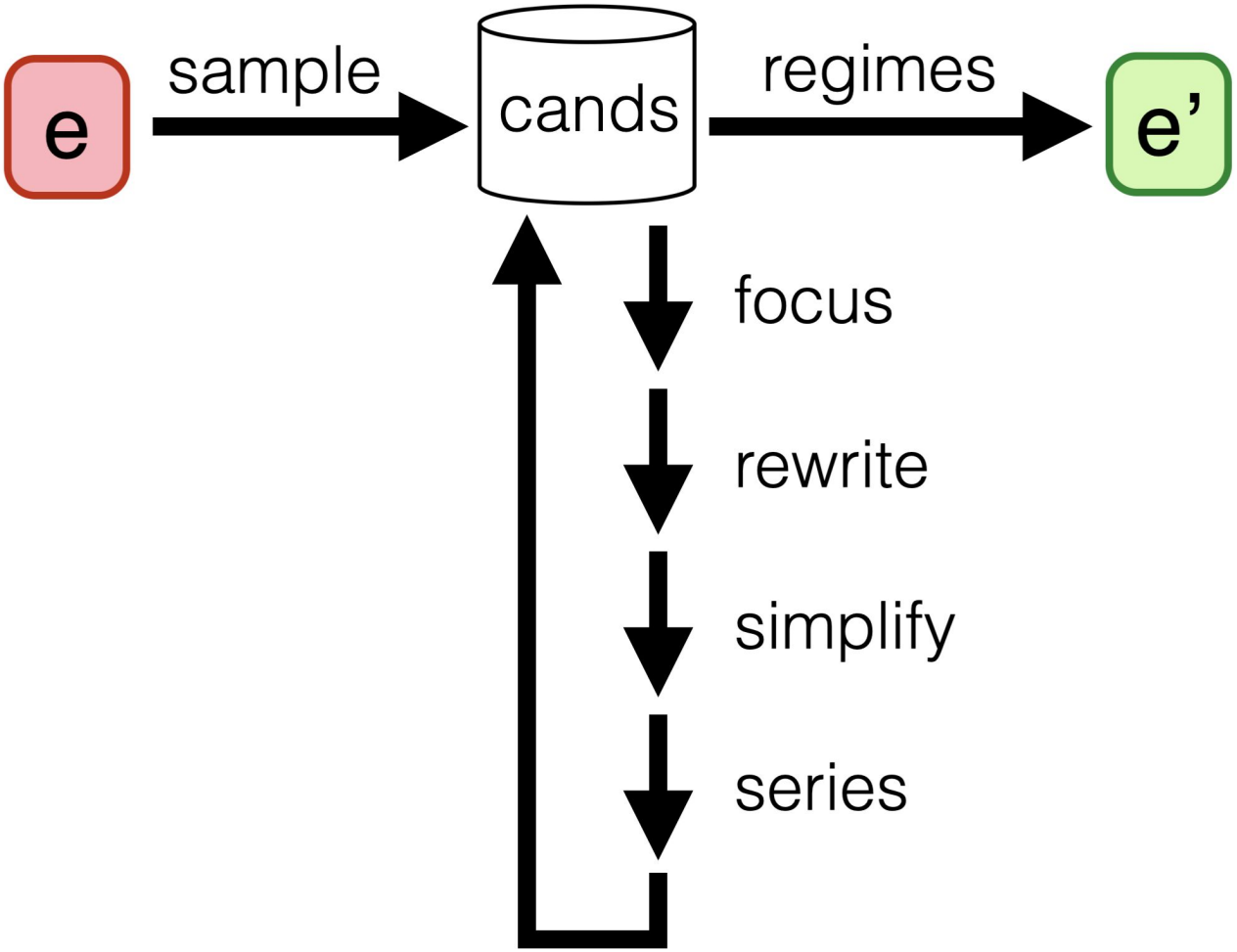

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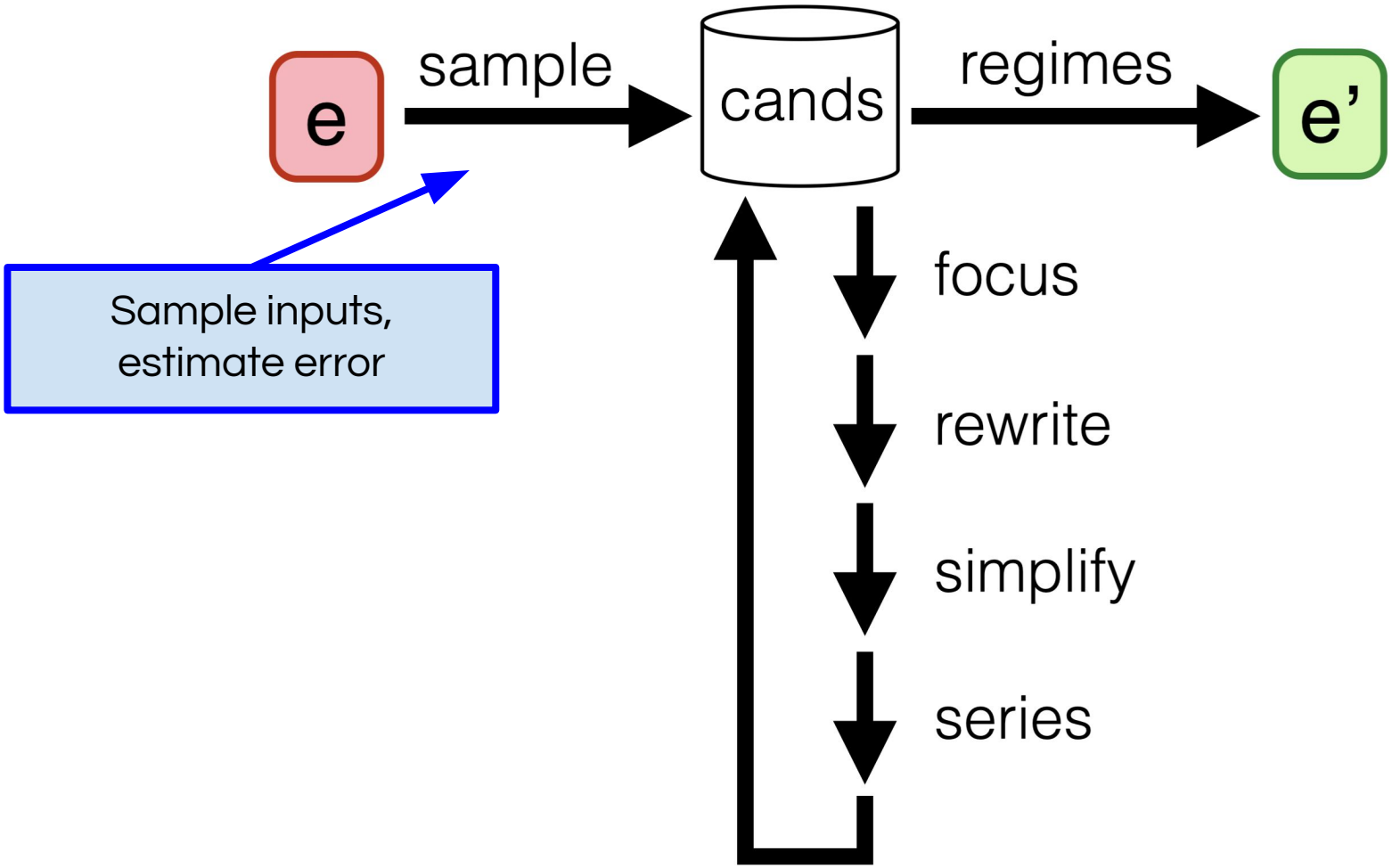

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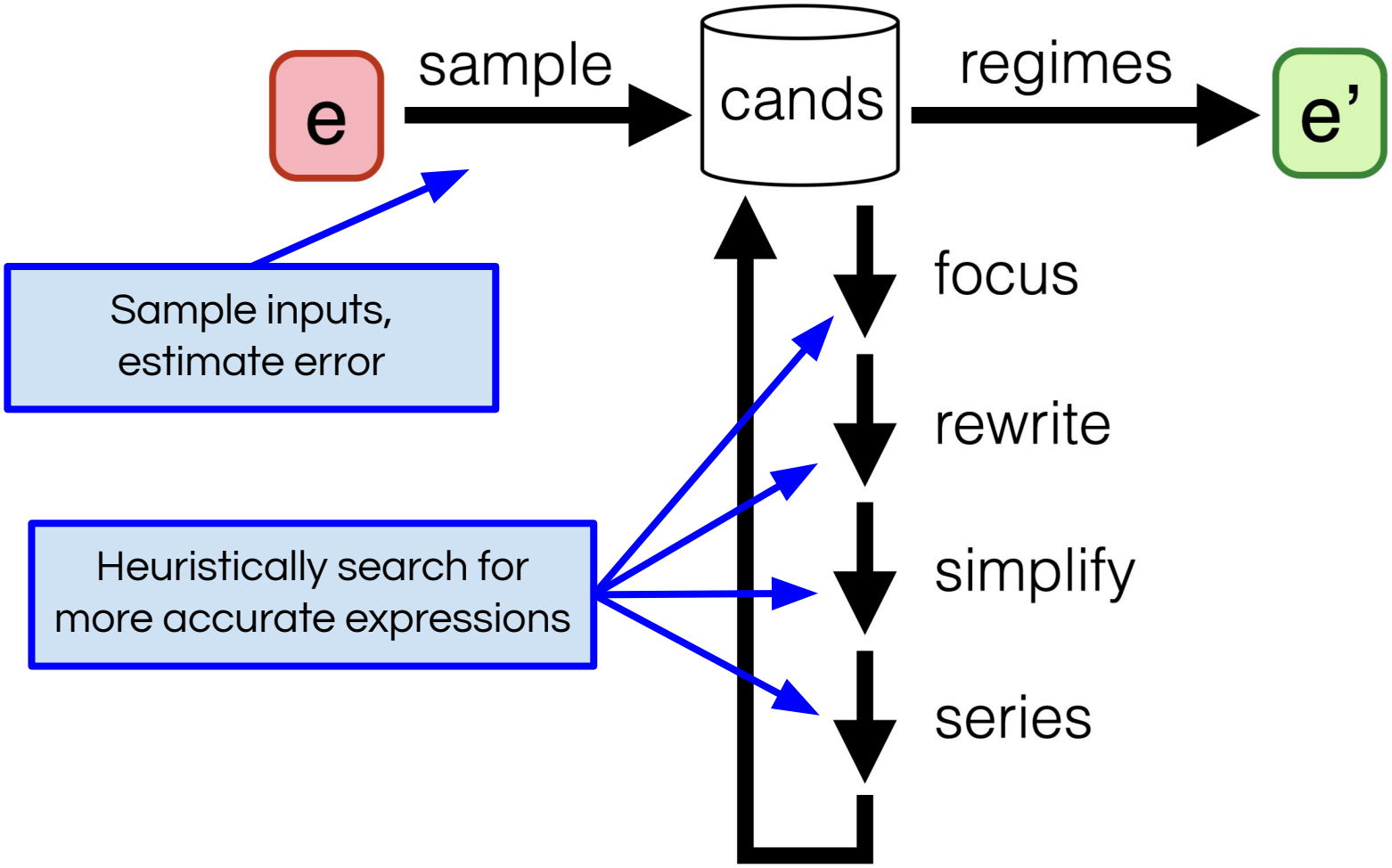

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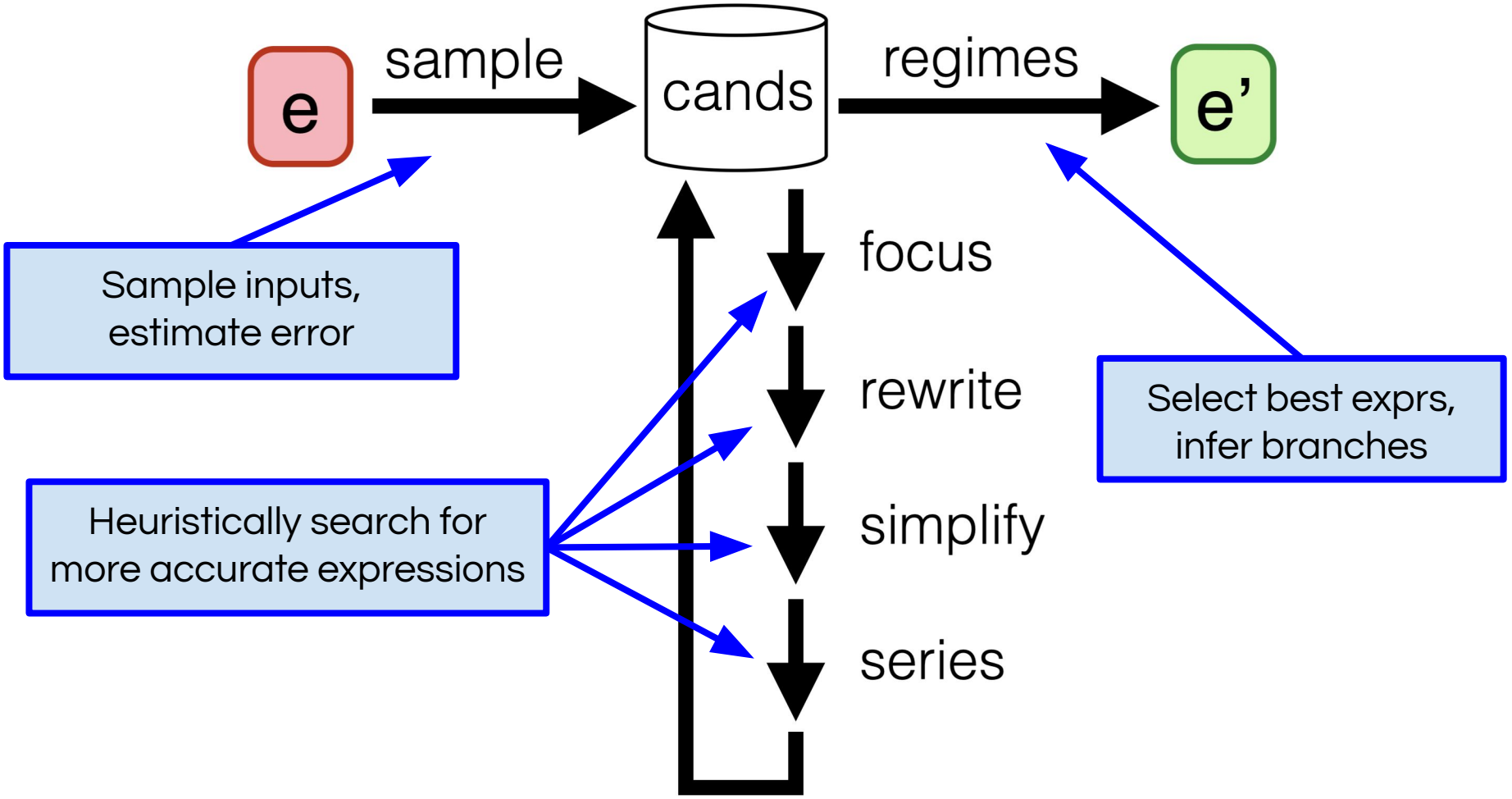

1. Trust

Why should users trust Herbie?

$$\frac{(-b) + \sqrt{b \cdot b - 4 \cdot (a \cdot c)}}{2 \cdot a}$$

$$\downarrow$$

$$\textbf{if } b \leq -9.877954748696237 \cdot 10^{+117} : \\ \quad 1 \cdot \left(\frac{c}{b} - \frac{b}{a}\right) \\ \textbf{elif } b \leq 9.997495995248066 \cdot 10^{-153} : \\ \quad \frac{1}{\frac{a}{\sqrt{b \cdot b - 4 \cdot (c \cdot a)} - b}} \cdot 2 \\ \textbf{else} : \\ \quad \frac{c}{b} \cdot -1$$

e’

Select best exprs, infer branches

Unpredictable, mysterious!

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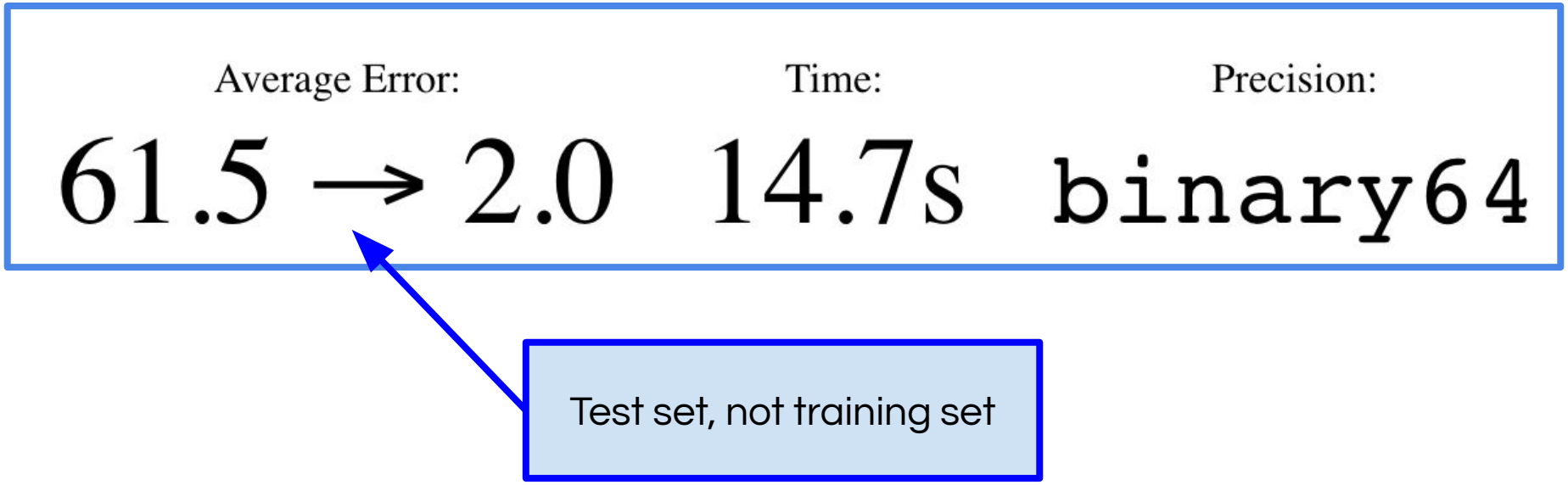

1. Trust

Why should users trust Herbie?

Split input into 3 regimes

Derivation

`if b < -9.87795474869623693e117`

Initial program
51.1

$$\frac{(-b) + \sqrt{b \cdot b - 4 \cdot (a \cdot c)}}{2 \cdot a}$$

Simplified
51.1

$$\rightsquigarrow \frac{\sqrt{b \cdot b - 4 \cdot (a \cdot c)} - b}{a \cdot 2}$$

Taylor expanded around -inf
2.4

$$\rightsquigarrow 1 \cdot \frac{c}{b} - 1 \cdot \frac{b}{a}$$

Simplified
2.4

$$\rightsquigarrow 1 \cdot \left(\frac{c}{b} - \frac{b}{a}\right)$$

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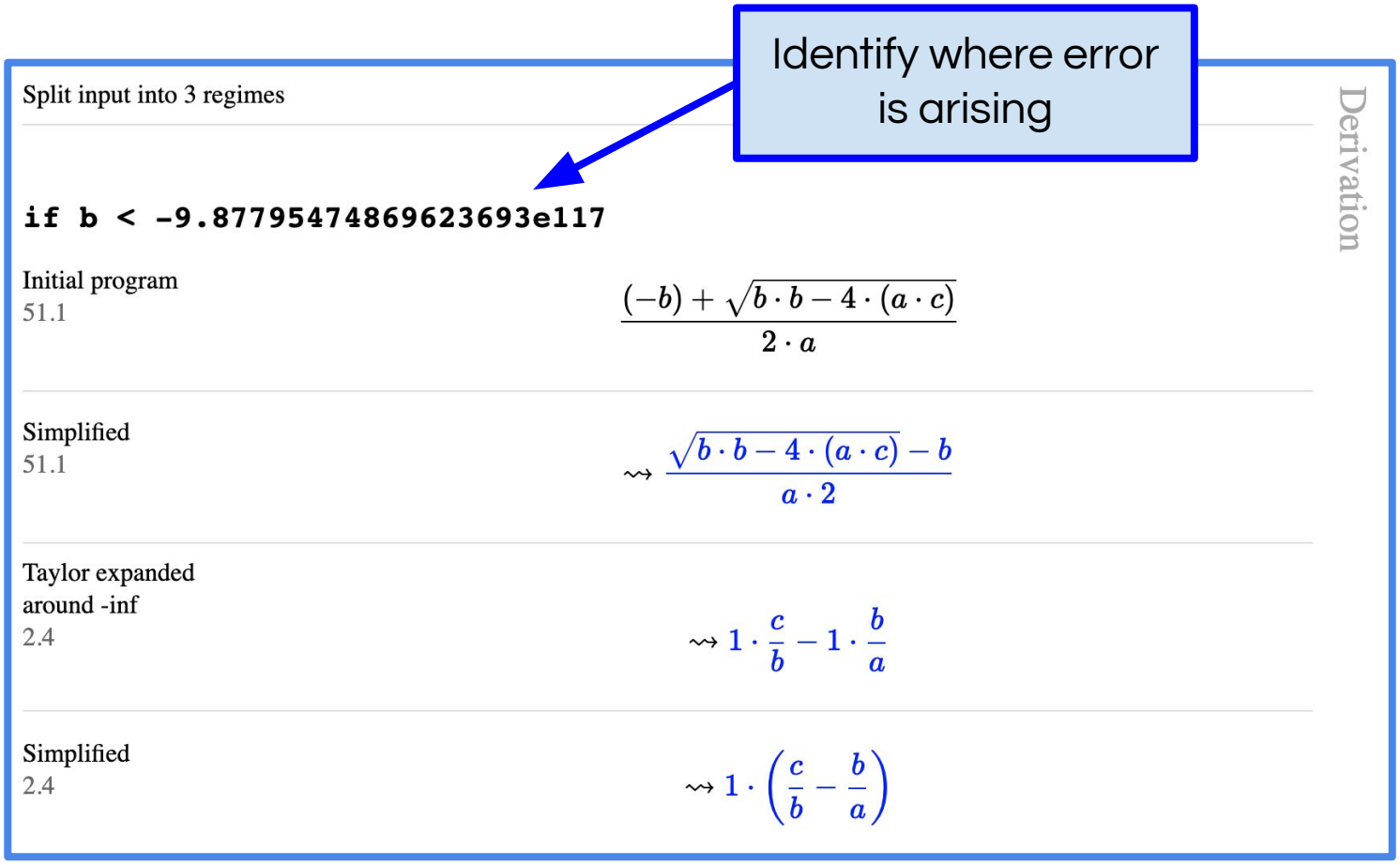

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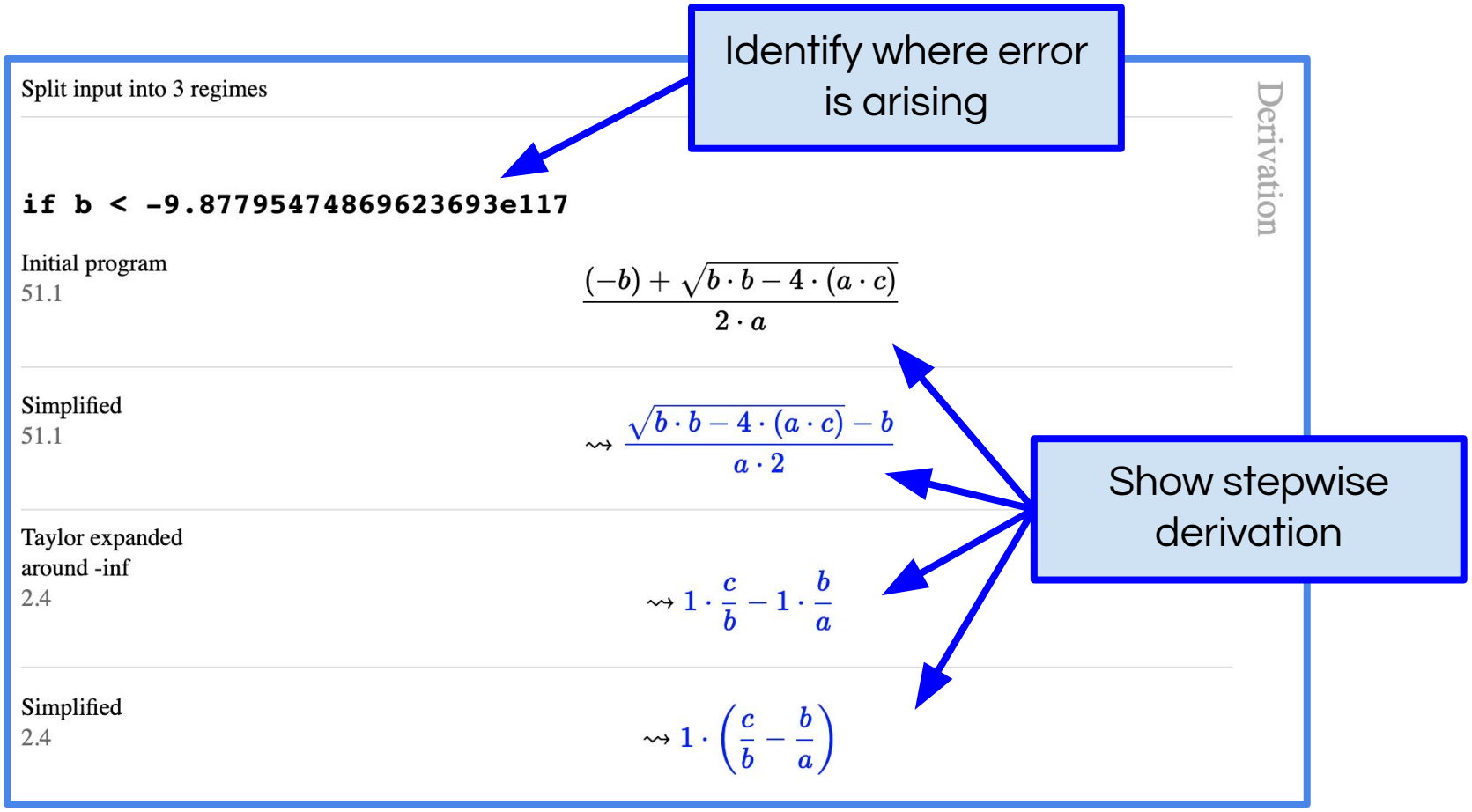

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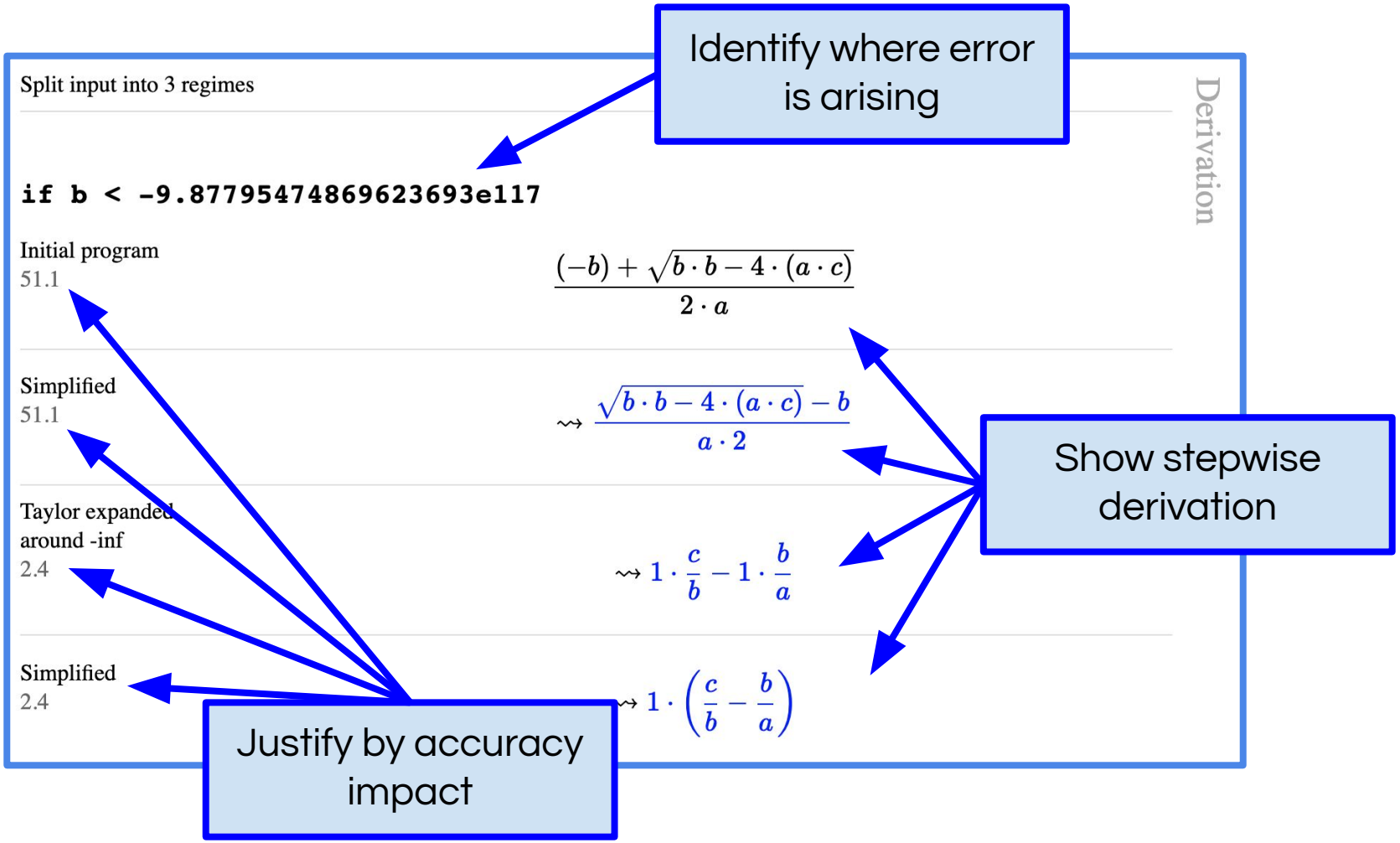

2. Measurement

That which is measured, improves.

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Herbie is a complex pipeline.

Each component individually robust.

- *Actually works against debugging!!*

Need to have per-module specs and metrics.

2. Measurement

That which is measured, improves.

```
5.164 * [progress]: [Phase 1 of 3] Setting up.
0.001 * * [progress]: [1/2] Preparing points
0.080 * * [progress]: [2/2] Setting up program.
0.083 * [progress]: [Phase 2 of 3] Improving.
0.083 * [simplify]: Simplifying using  #<procedure:simplify-batch-egg> :
   (- (exp (* a x)) 1.0)
0.137 * * [simplify]: iteration  0 :  4778  enodes  (cost  6 )
0.137 * * [simplify]: iteration  1 :  4778  enodes  (cost  6 )
0.137 * [simplify]: Simplified to:
   (- (pow (exp a) x) 1.0)
0.141 * * [progress]: iteration 1 / 4
0.141 * * * [progress]: picking best candidate
0.146 * * * * [pick]: Picked  #<alt (Î» (a x) (- (exp (* a x)) 1.0))>
0.146 * * * [progress]: localizing error
0.151 * * * [progress]: generating rewritten candidates
0.151 * * * * [progress]: [ 1 / 2 ] rewriting at (2)
0.155 * * * * [progress]: [ 2 / 2 ] rewriting at (2 1)
0.157 * * * [progress]: generating series expansions
0.157 * * * * [progress]: [ 1 / 2 ] generating series at (2)
0.157 * [approximate]: Taking taylor expansion of (- (exp (* a x)) 1.0) in (a x) around 0
0.157 * [taylor]: Taking taylor expansion of (- (exp (* a x)) 1.0) in x
0.157 * [taylor]: Taking taylor expansion of (exp (* a x)) in x
0.157 * [taylor]: Taking taylor expansion of (* a x) in x
0.157 * [taylor]: Taking taylor expansion of a in x
0.157 * [taylor]: Taking taylor expansion of x in x
0.157 * [taylor]: Taking taylor expansion of 1.0 in x
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0.157 * [taylor]: Taking taylor expansion of 0 in x
0.158 * [taylor]: Taking taylor expansion of x in x
0.158 * [taylor]: Taking taylor expansion of (* 1/2 (pow x 2)) in x
0.158 * [taylor]: Taking taylor expansion of 1/2 in x
0.158 * [taylor]: Taking taylor expansion of (pow x 2) in x
0.158 * [taylor]: Taking taylor expansion of x in x
0.158 * [taylor]: Taking taylor expansion of (* 1/6 (pow x 3)) in x
0.158 * [taylor]: Taking taylor expansion of 1/6 in x
0.158 * [taylor]: Taking taylor expansion of (pow x 3) in x
0.158 * [taylor]: Taking taylor expansion of x in x
0.159 * [taylor]: Taking taylor expansion of (* 1/24 (pow x 4)) in x
```
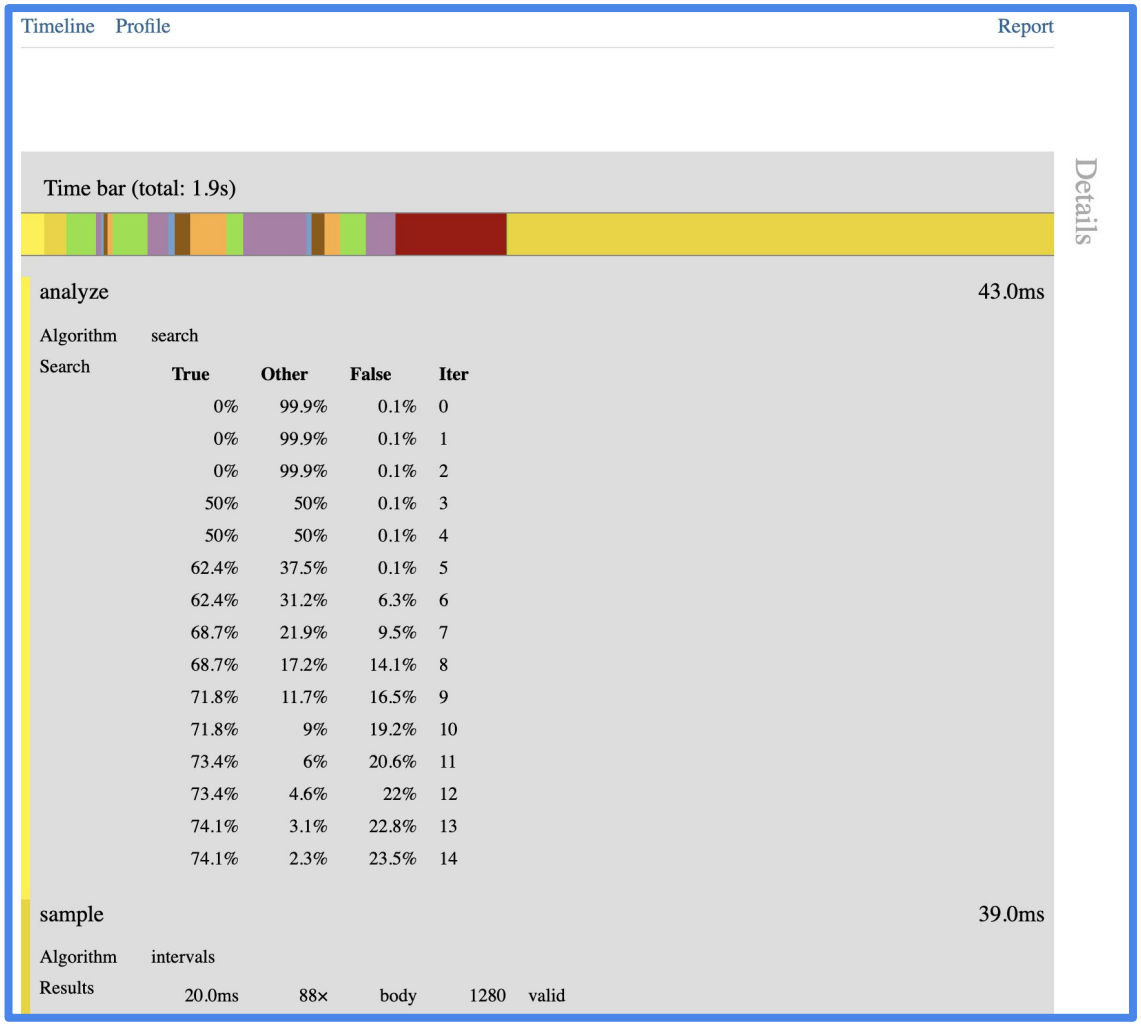

No “just enough” level of detail is right for logs…

2. Measurement

That which is measured, improves.



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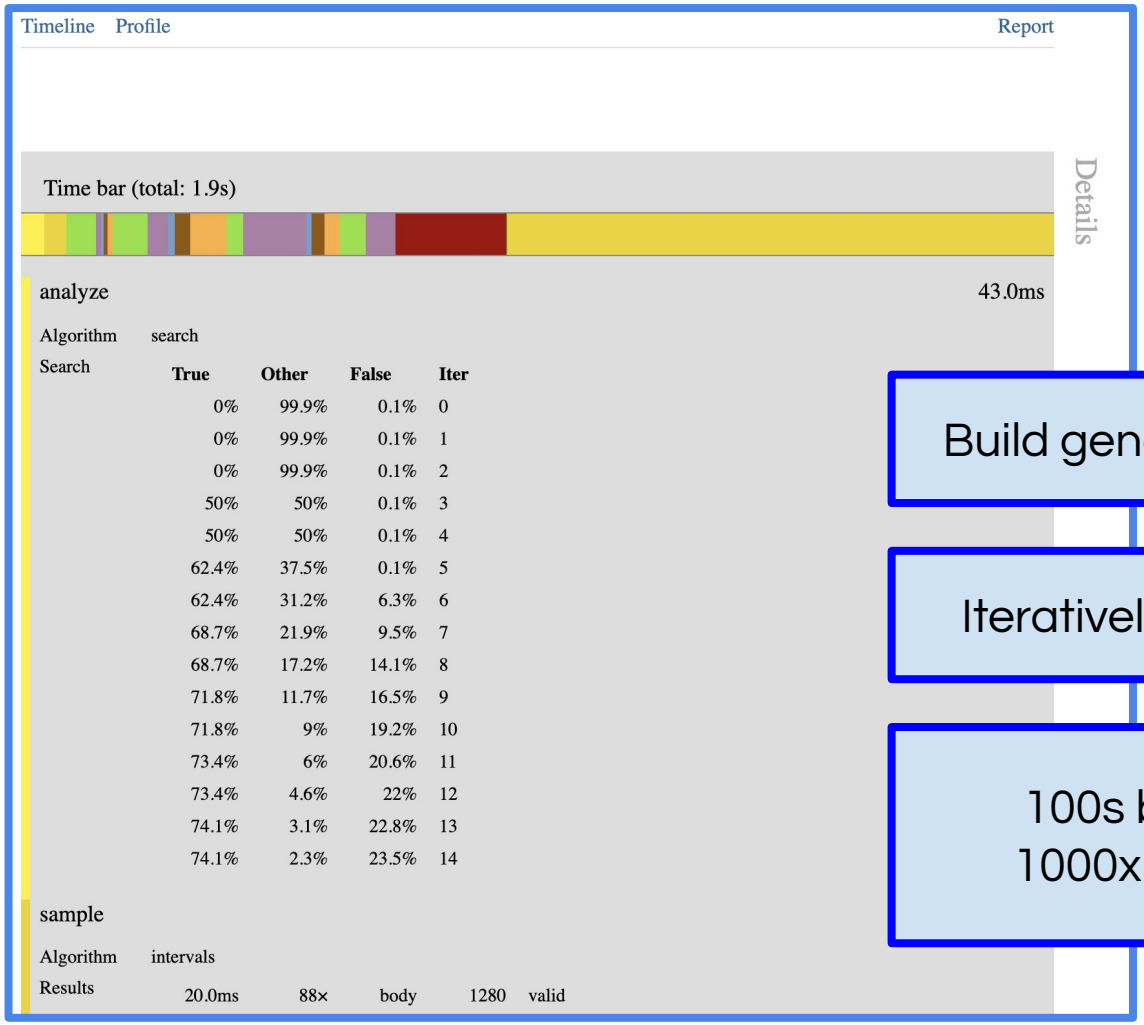

Build generic telemetry!

Iteratively develop viz!

100s bugs fixed,
1000x speedups!

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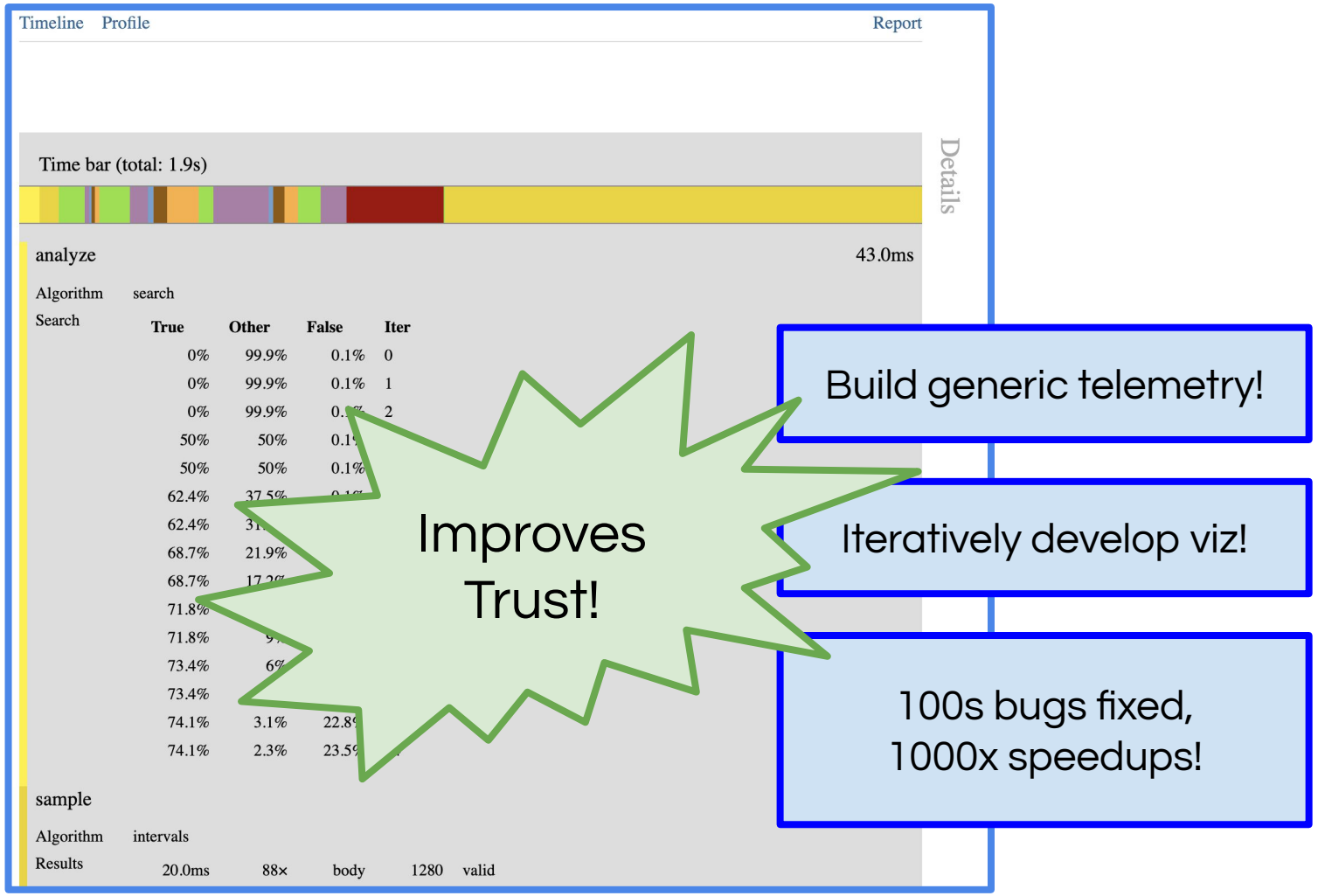

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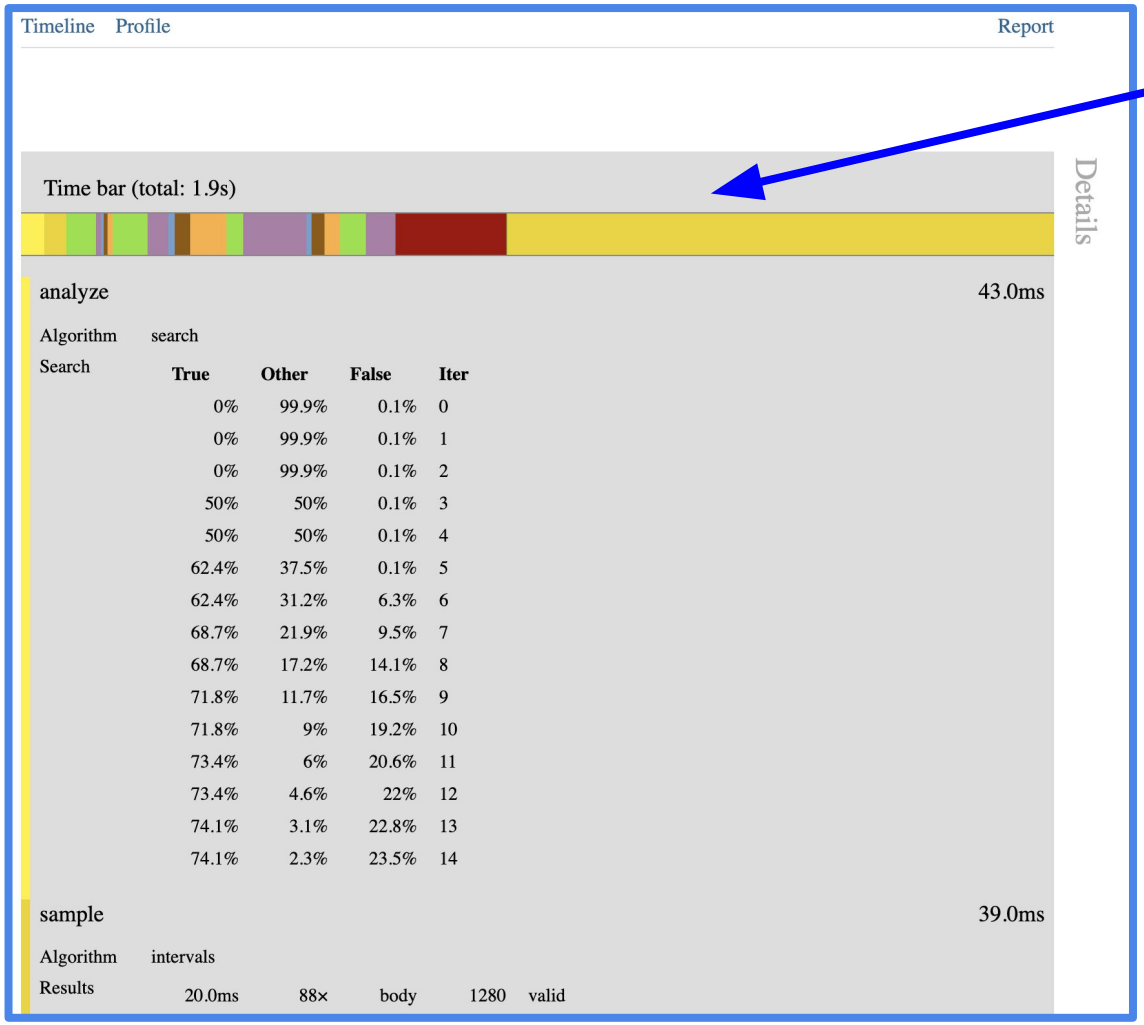

Global overviews

2. Measurement

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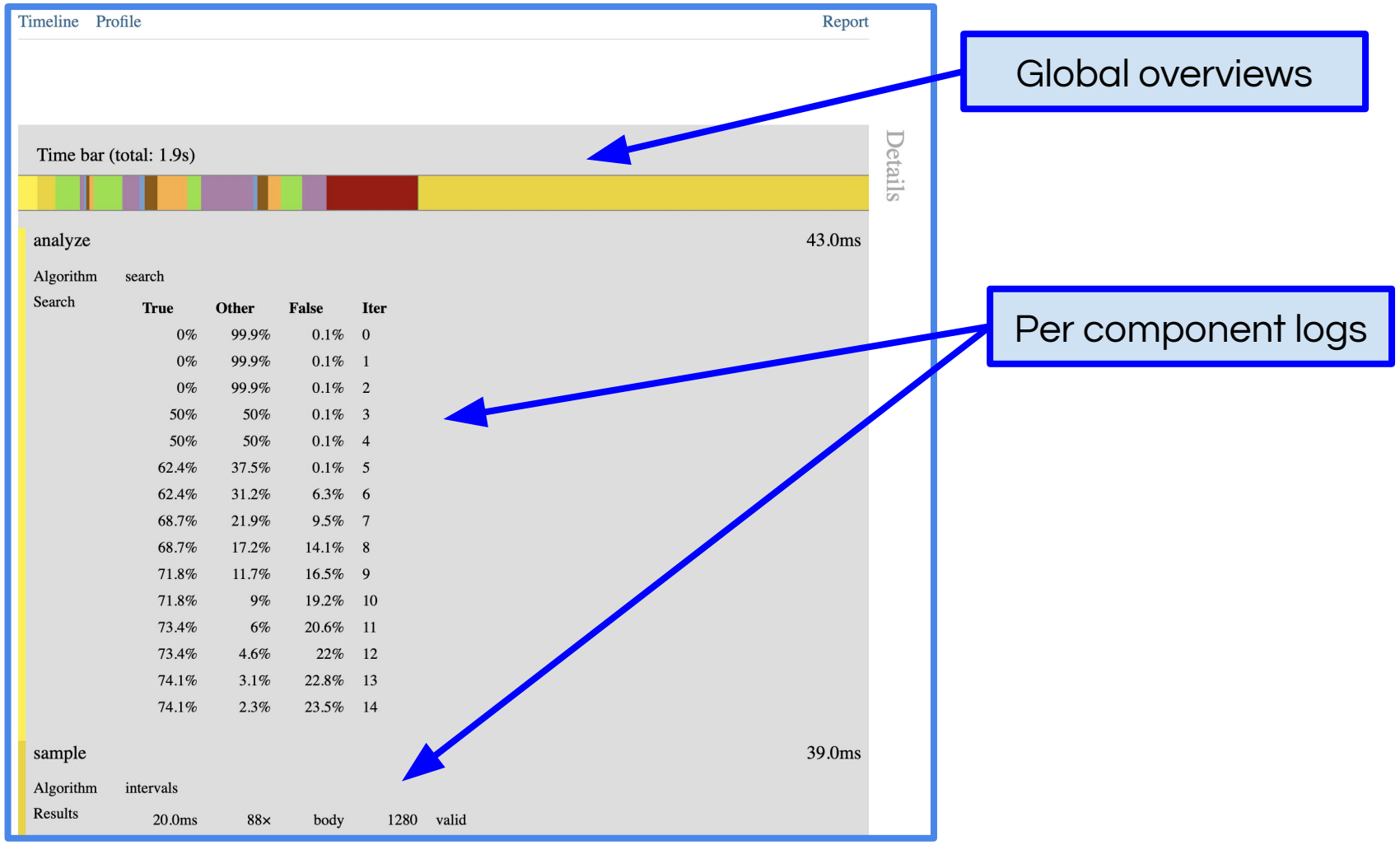

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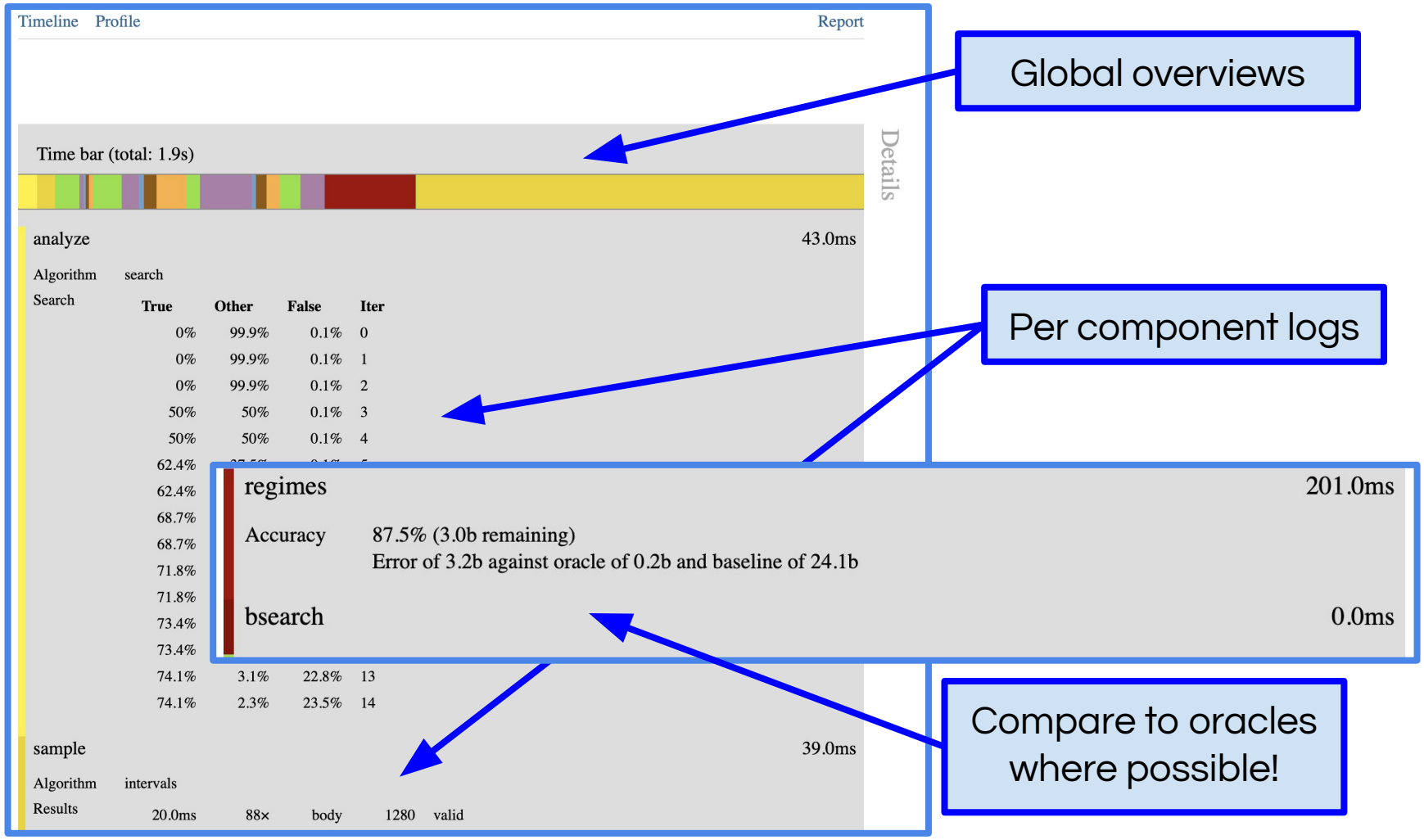

3. Community

We are all in this together.

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Our thinking in 2016

Herbie is just one piece of the puzzle.

The workbench demands cooperating tools:

- Rigorous error bounds, precision tuning, performance optimization, …

How should they interact? Need standards!

3. Community

We are all in this together.

Our thinking in 2016

Herbie is just one piece of the puzzle.

The workbench demands cooperating tools:

Hov

Numerics research is a community effort.

We need more open communications!

We need more collaboration opportunities!

We need more shared infrastructure!

View today

3. Community

We are all in this together.

FPTalks 2020

The leading edge of floating-point research

Home Benchmarks Compilers Standards

The floating-point research community.

Online. June 24, 2020.

FPTalks 2020 was held online over Zoom. All talks were live-streamed and recorded on Youtube. Each talk was 10 minutes long, followed by audience questions.

ada
Applications Driving Architectures

FPTalks 2020 was supported in part by the Applications Driving Architectures (ADA) Research Center, a JUMP Center co-sponsored by SRC and DARPA.

Session 1 (Session Stream) **8:00–9:00 PDT**

Welcome

Pavel Panchekha, University of Utah

Video, Slides, FPBench

3. Community

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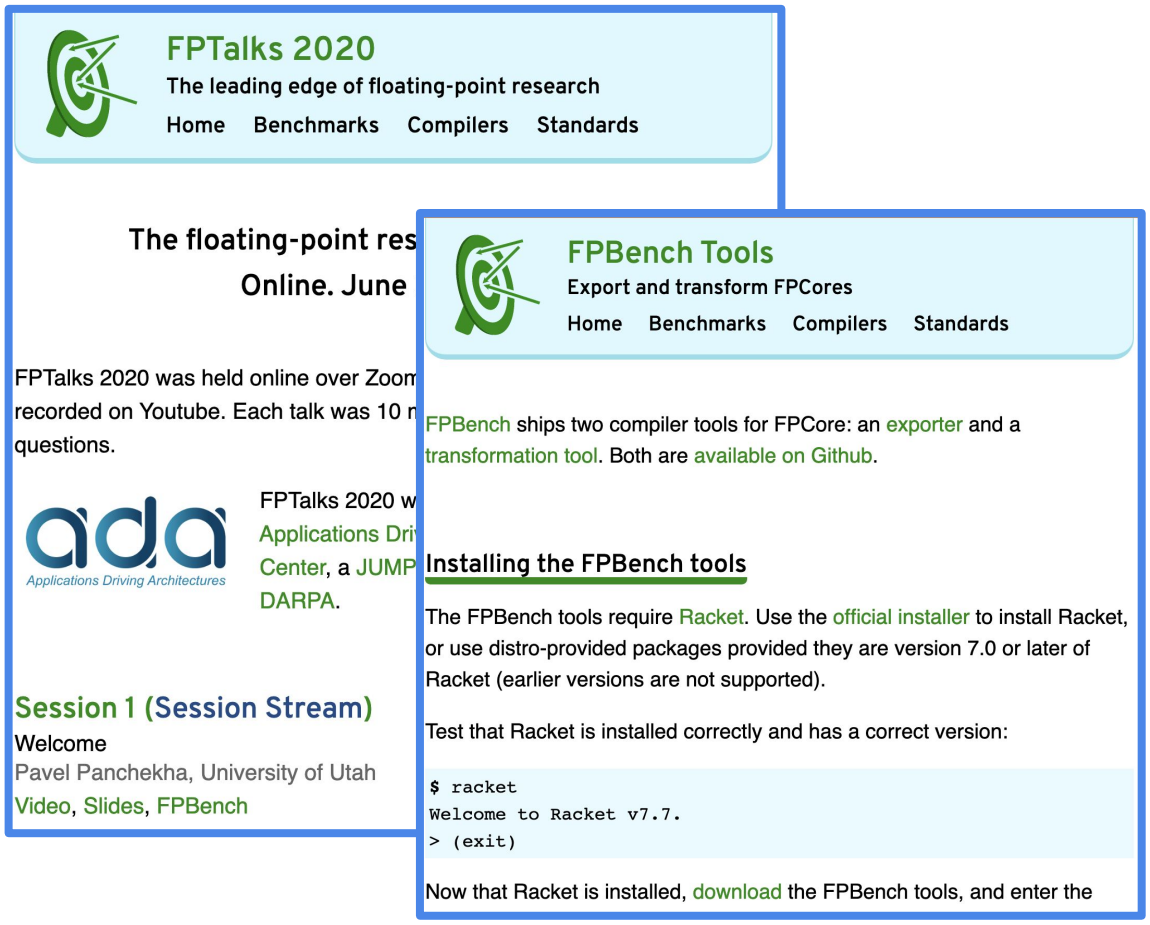

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FPBench Tools

Export and transform FPCores

Home Benchmarks Compilers Standards

FPBench ships two compiler tools for FPCore: an exporter and a transformation tool. Both are available on Github.

Installing the FPBench tools

The FPBench tools require Racket. Use the official installer to install Racket, or use distro-provided packages provided they are version 7.0 or later of Racket (earlier versions are not supported).

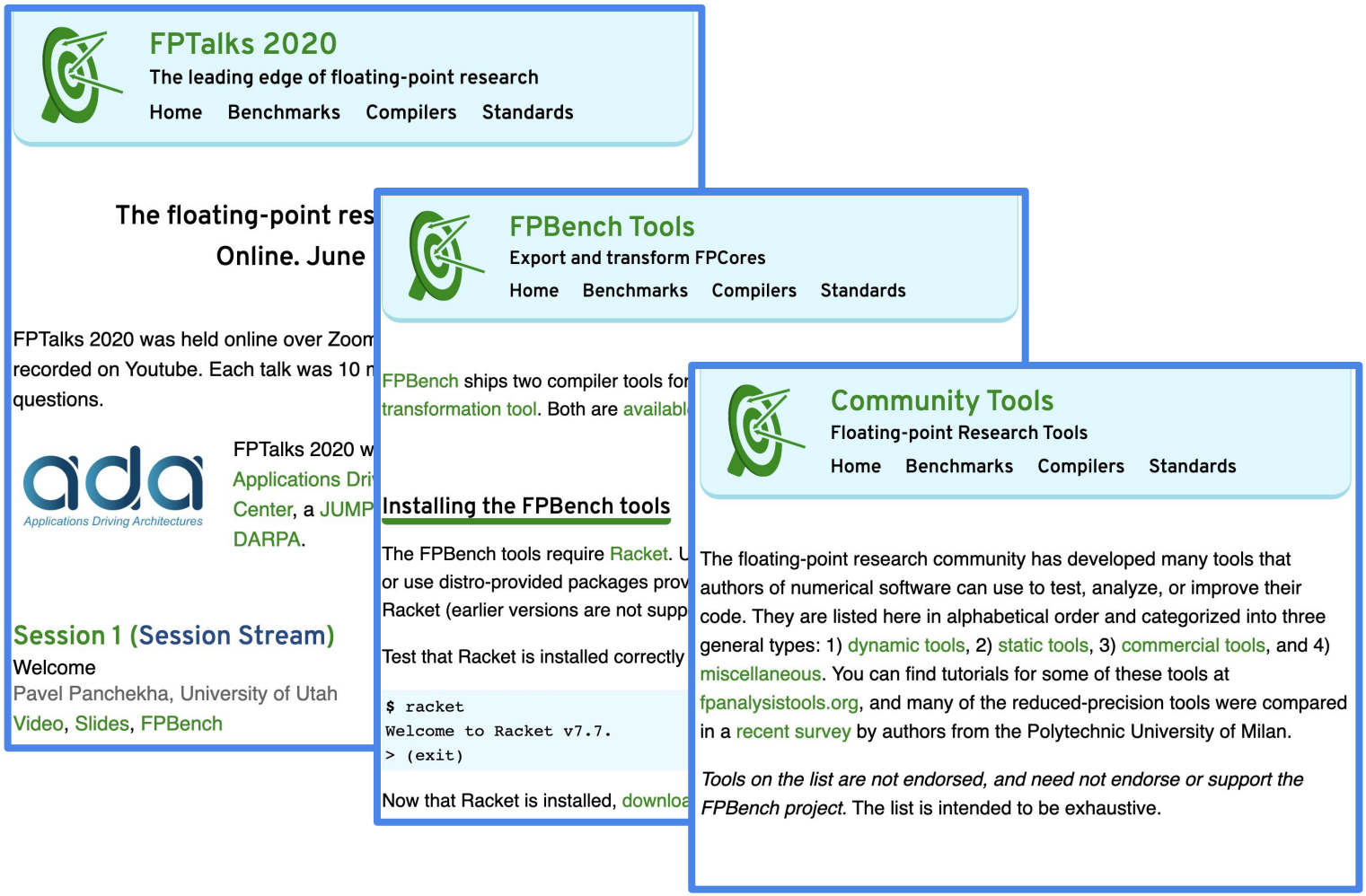
Test that Racket is installed correctly and has a correct version:

```
$ racket
Welcome to Racket v7.7.
> (exit)
```

Now that Racket is installed, download the FPBench tools, and enter the

3. Community

We are all in this together.



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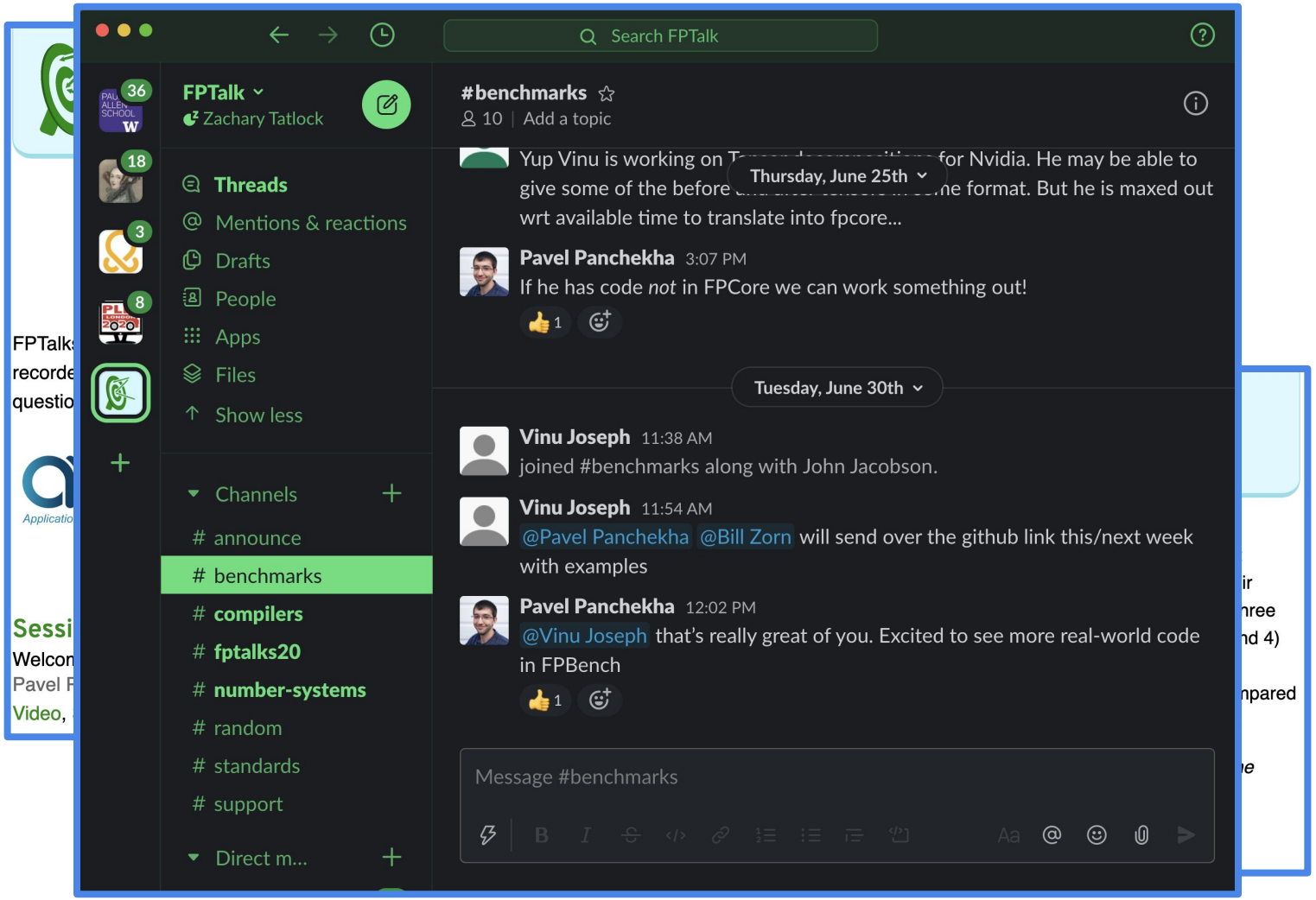

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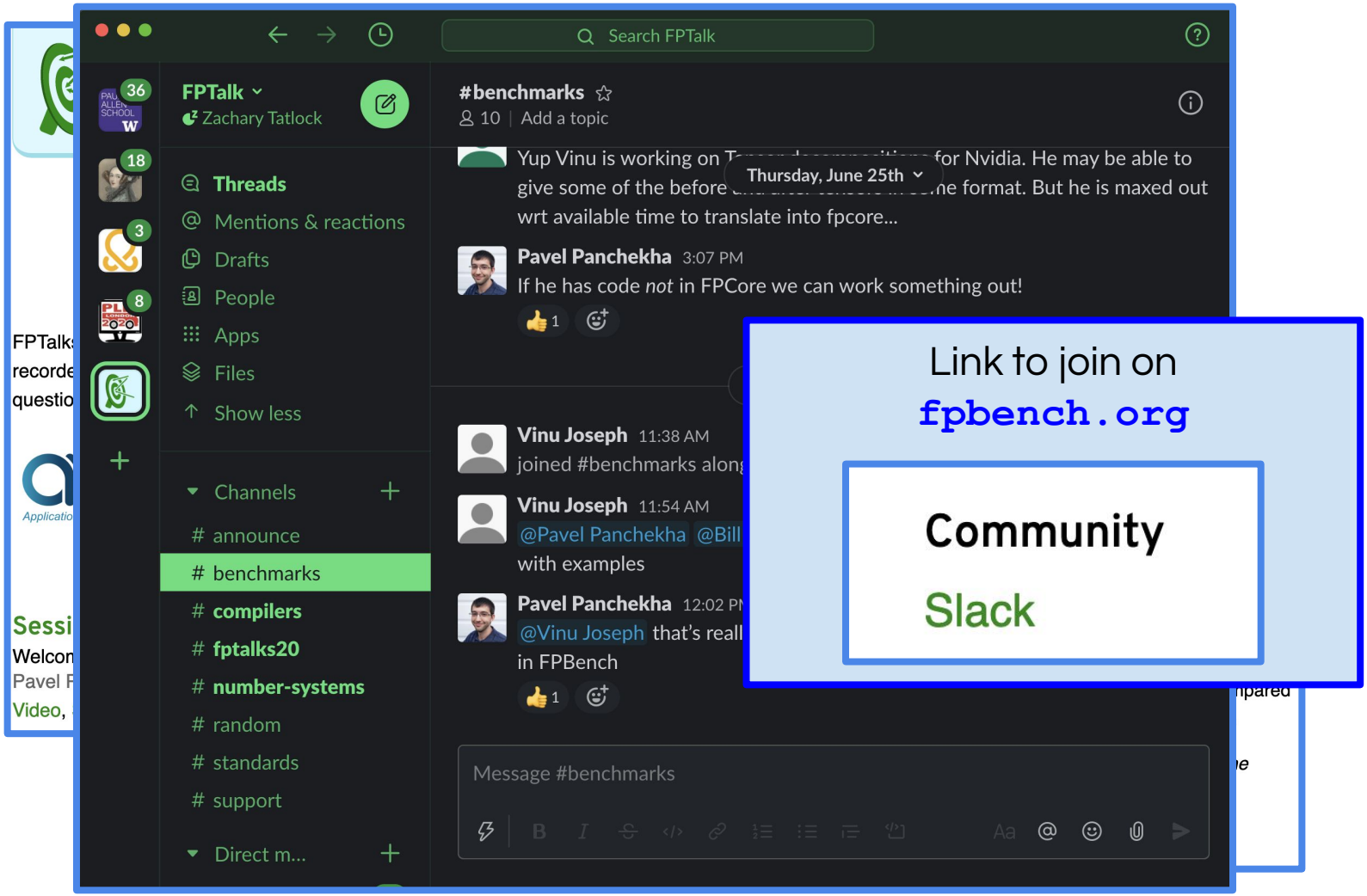

4. Generality

Supporting a rapidly evolving landscape

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Explosion of numerical representations.

How do we adapt code? Optimize? Need tools!

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Supporting a rapidly evolving landscape

Explosion of numerical representations.

How do we adapt code? Optimize? Need tools!

```
(FPCore (x)
 :name "float32 ulp"
 :precision binary64
  (! :precision binary32 (- (! :round toPositive (cast x ))
                            (! :round toNegative (cast x)))))
```

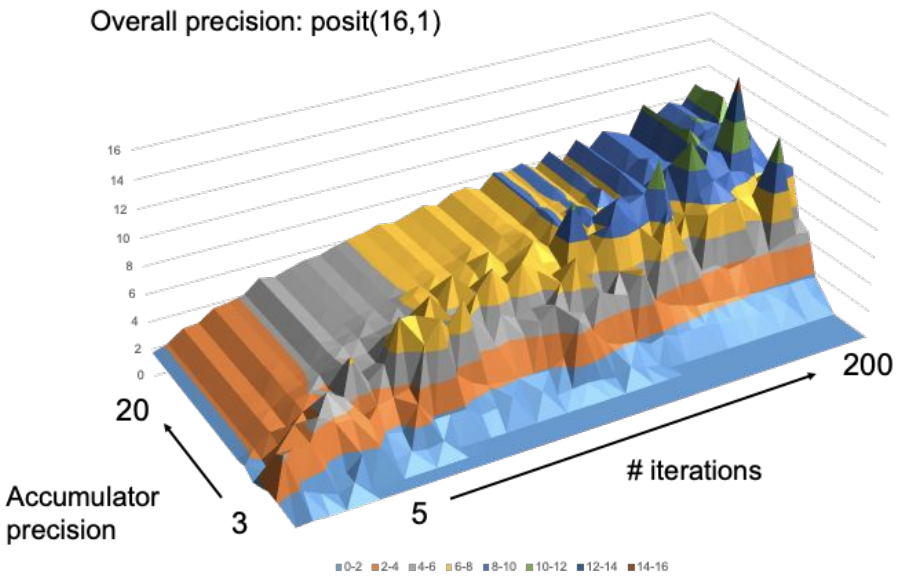
```
(FPCore (x)
 :name "round with addition"
 :spec (round x)
  (let ([n (! :precision binary64 6755399441055744)])
    (! :precision binary64 :round nearestEven (- (+ x n) n))))
```

"Precision scopes" specify MPMF!

4. Generality

Supporting a rapidly evolving landscape

```
(FPCore (n)
 :name "arclength"
 :cite (precimonious-2013)
 :precision (posit 1 16)
 :pre (>= n 0)

 (let* ([dppi (acos -1)]
        [h (/ dppi n)]
        [t1 0])

  (while* (<= i n)

   ([t2 0
     (let ([x (* i h)])
      (while* (<= k 5)
       ([d1 1
         (! :precision binary32 (* d1 2))]
        [t1 x
         (+ t1 (/ (sin (* d1 x)) d1))]
        [k 1 (+ k 1)])
       t1))]

    [s1 0
     (let ([s0 (sqrt (+ (* h h) (* (- t2 t1) (- t2 t1))))])
      (! :precision (fixed -8 12) (+ s1 s0)))]

    [t1 t1 t2]

    [i 1 (+ i 1)])

   s1)))
```



Enables parameter sweeps: iters, cost, accuracy

4. Generality

Supporting a rapidly evolving landscape

Increasing interest in numerics of tensor code.

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Supporting a rapidly evolving landscape

Increasing interest in numerics of tensor code.

```
(FPCore matmul ((A am an) (B bm bn))
   :pre (== an bm)
    (tensor ([m am]
             [n bn])
    (for ([i bm])
         ([prod 0 (+ prod (* (ref A m i) (ref B i n)))])
         prod))
  )
```

FPBench 2.0 adds tensor support, keeps minimal core

Where do we go from here?

Trust : ground truth for MPMF

Measurement : metrics for tensor benchmarks

Community : outreach from research to practice

Generality : extensibility for new formats

A brief announcement...

Herbie 1.4 just released!

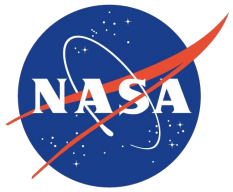
Please take it for a spin :)

herbie.uwplse.org

Many Acknowledgements!

Alex Sanchez-Stern
Bill Zorn
David Thien
Oliver Flatt
Brett Saiki
Jason Qiu
Ian Briggs

Heiko Becker
Eva Darulova
Max Willsey
James Wilcox
Jack Firth
Mike Lam
… and more!




Sandia National Laboratories


Thank You!

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herbie.uwplse.org

fpbench.org